

For all questions, answer choice "E. NOTA" means none of the above answers is correct. Also, "DNE" means "does not exist".

1. Which of these series represents $\int e^{x^2} dx$?

A. $C + \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$ B. $C + \sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$ C. $C + \sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!(2n+1)}$ D. $C + \sum_{n=1}^{\infty} \frac{x^{2n+1}}{n!(2n+1)}$

E. NOTA

2. $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\sum_{k=0}^{\infty} \left(-2 + \frac{1}{n}\right)^k + \sum_{k=0}^{\infty} \left(-2 + \frac{2}{n}\right)^k + \sum_{k=0}^{\infty} \left(-2 + \frac{3}{n}\right)^k + \cdots + \sum_{k=0}^{\infty} (-1)^k \right]$

A. $\ln\left(\frac{1}{3}\right)$ B. $\ln\left(\frac{1}{2}\right)$ C. $\ln\left(\frac{2}{3}\right)$ D. $\ln\left(\frac{3}{2}\right)$ E. NOTA

3. Under which of the following conditions on the series $\sum_1^{\infty} a_n$ with positive terms is the Ratio Test inconclusive?

A. $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} > 1$ B. $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} < 1$ C. $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = 1$

D. Both A and C

E. NOTA

4. What is the interval of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(3x-5)^n}{n}$?

A. $\left[-\frac{1}{3}, \frac{1}{3}\right]$ B. $\left(-\frac{1}{3}, \frac{1}{3}\right)$ C. $\left(\frac{4}{3}, 2\right]$ D. $\left[\frac{4}{3}, 2\right)$ E. NOTA

5. Series A is $\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$ and series B is $\sum_{n=1}^{\infty} \frac{(-3)^n}{(2n)n}$. Which of the following is true?

A. Series A and B are conditionally convergent

B. Series A converges absolutely and B converges conditionally

C. Series A converges conditionally and B diverges

D. Series A converges absolutely and B diverges

E. NOTA

6. Which of the following is the nth degree Maclaurin series for the function $f(x) = \cos(\sqrt{x})$?

A. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ B. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$ C. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$ D. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ E. NOTA

7. Using the Maclaurin series for $f(x) = 4\arctan(x)$ evaluated at $x = 1$, what degree is required to obtain an approximation of π that is accurate up to the thousandths place?

A. 1

B. 3

C. 5

D. 7

E. NOTA

8. The power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n}$ represents which function?

A. $\ln(x)$

B. $\ln\left(\frac{1}{x}\right)$

C. $-\frac{1}{x}$

D. $\frac{1}{x}$

E. NOTA

9. Which of the following series is conditionally convergent?

A. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ B. $\sum_{n=1}^{\infty} \frac{(-2)^n}{n}$ C. $\sum_{n=1}^{\infty} \frac{(-2)^n}{3^n}$ D. $\sum_{n=1}^{\infty} \frac{(-4)^n}{n!}$

E. NOTA

10. Which of the following tests may be used to determine the convergence of $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$?

- I. Ratio Test
 II. Root Test
 III. Integral Test

A. I only B. II only C. III only D. I, II, and III E. NOTA

11. What is the sum of $1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \dots$

A. It diverges B. $\sqrt{5}$ C. e D. π E. NOTA

12. The 4th degree Taylor polynomial about $x = 1$ for the function $f'(x)$ is $P_4(x) = 2 + 5(x - 1) - 3(x - 1)^2 + 4(x - 1)^3 - (x - 1)^4$. Given this, what is $f'''(1) - f''(1)$?

A. -11 B. -8 C. 7 D. 30 E. NOTA

13. The 4th degree Taylor polynomial about $x = 2$ for $g(x)$ is $P_4(x) = 3 + 4x - 5x^2 + \frac{x^3}{4} - \frac{5x^4}{3}$. What is $g'''(2) - g''(2)$?

A. -9 B. 5.25 C. 8.5 D. 11.5 E. NOTA

14. What is the result when the Root Test and the Ratio Test are both applied to the following series:

$$\sum_{n=1}^{\infty} \frac{4^n}{3^n(n^7 + 6n^2)} ?$$

- A. Both conclude it diverges D. The Root Test shows convergence and the Ratio Test is inconclusive
 B. Both conclude it converges E. NOTA
 C. The Ratio Test shows divergence and the Root Test is inconclusive

15. Which series could be successfully used in a Direct Comparison Test to determine whether the following series converges or diverges: $\sum_{n=1}^{\infty} \frac{n!}{5^n(n^3+2)}$?

A. $\sum_{n=1}^{\infty} \frac{n!}{5^n}$ B. $\sum_{n=1}^{\infty} \frac{n!}{(n^3+2)}$ C. $\sum_{n=1}^{\infty} \frac{4^n}{5^n(n^3+2)}$ D. $\sum_{n=1}^{\infty} \frac{6^n}{5^n(n^3+2)}$ E. NOTA

16. Alice and Bob consider the terms of the alternating harmonic series: $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$. Both decide to add up the terms in the manner and order each finds most convenient. Alice gets a sum of $\ln(2)$ while Bob gets half of that as the sum. What is the explanation for this?

- A. Alice has the correct sum; Bob must have made some error
- B. Bob has the correct sum; Alice must have made sum error
- C. Both must have made some error, the correct sum is a different amount
- D. Both could be correct; the value depends on the order in which the terms are added
- E. NOTA

17. Which of the following is true about the sequence $a_n = \left(1 + \frac{1}{n}\right)^n$?

- A. It converges to 1
- B. It converges to e
- C. It converges to π
- D. It diverges
- E. NOTA

18. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^n}{5 \cdot 3^n}$

- A. $\frac{2}{25}$
- B. $\frac{2}{5}$
- C. $\frac{3}{5}$
- D. 3
- E. NOTA

19. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$

- A. $\frac{6}{5}$
- B. $\frac{5}{4}$
- C. $\frac{4}{3}$
- D. $\frac{3}{2}$
- E. NOTA

20. Consider the recursively defined sequence $a_n = \sqrt{2 + a_{n-1}}$ with $a_1 = \sqrt{3}$. What is $\lim_{n \rightarrow \infty} a_n$?

- A. -1
- B. 2
- C. $\sqrt{6}$
- D. D.N.E.
- E. NOTA

21. A square has sides of length 16. A smaller square created inside it by connecting the midpoints of all the sides and one of the four triangles created by this process is shaded. The process is repeated with the smaller square, but this time three of the four triangles created are shaded. Again the process is repeated and only one triangle is shaded, then again with three triangles, and so on. If this continues indefinitely, what is the total shaded area?

- A. $\frac{32}{3}$
- B. 16
- C. 32
- D. 128
- E. NOTA

22. For what values of k will the following series converge: $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^k}$

- A. $k > 0$
- B. $k \geq 1$
- C. $k > 1$
- D. $k \geq 2$
- E. NOTA

23. How many terms of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+1}$ would be required to obtain an approximation within .01 of the full sum?

- A. 7
- B. 8
- C. 9
- D. 10
- E. NOTA

24. Find the sum of $-\sum_{n=0}^{\infty} \frac{(-4)^{n+1}}{(2n+1)!}$

- A. $\frac{1}{2} \sin(2)$ B. $\frac{1}{2} \sin(4)$ C. $\sin(2)$ D. $\sin(4)$ E. NOTA

25. If $a_n, b_n > 0 \forall n$ and $\lim_{n \rightarrow \infty} \frac{b_n}{a_n} = \frac{1}{3}$. Which of the following MUST be true?

- A. $\sum_{n=1}^{\infty} a_n$ converges B. $\sum_{n=1}^{\infty} b_n$ converges C. Both series converge
D. Both series diverge E. NOTA

For each of the questions in #26-30 you will be given a pair of series, A and B. The answer choices will always be as follows (use E if one or both converge conditionally):

- A. A converges absolutely and B diverges B. B converges absolutely and A diverges
C. Both converge absolutely D. Both diverge E. NOTA

26. A: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n-2)}{n(n+5)}$ B: $\sum_{n=1}^{\infty} \left(\frac{10}{9}\right)^n$

- A. A converges absolutely and B diverges B. B converges absolutely and A diverges
C. Both converge absolutely D. Both diverge E. NOTA

27. A: $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$ B: $\sum_{n=1}^{\infty} \frac{(n^3+5n)10^{n+2}}{n!}$

- A. A converges absolutely and B diverges B. B converges absolutely and A diverges
C. Both converge absolutely D. Both diverge E. NOTA

28. A: $\sum_{n=1}^{\infty} (-1)^{n+1} n^{(-\frac{4}{3})}$ B: $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n^2}$

- A. A converges absolutely and B diverges B. B converges absolutely and A diverges
C. Both converge absolutely D. Both diverge E. NOTA

29. A: $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$ B: $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n+1}\right)^n$

- A. A converges absolutely and B diverges B. B converges absolutely and A diverges
C. Both converge absolutely D. Both diverge E. NOTA

30. A: $\sum_{n=1}^{\infty} \frac{3^{2n}(\frac{1}{4} + \arctan(n))}{7^{n-1}}$ B: $\sum_{n=1}^{\infty} \frac{8\cos(n)}{n\sqrt{n}}$

- A. A converges absolutely and B diverges B. B converges absolutely and A diverges
C. Both converge absolutely D. Both diverge E. NOTA